Intrinsic Verification and a Heat Conduction Database

Kevin D. Cole and Keith A. Woodbury
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Introduction

In the area of heat conduction and diffusion, many analytical solutions are available already in classic books on the subject. The authors of such books have assumed that the mathematical expression is all the reader needs, when in fact for engineering applications, considerable additional work is usually required in order to obtain numerical values. Analytical solutions could be compared with imported furniture that comes packed flat in a box – they should come with instructions and with tools needed to assemble them into a usable form. In this paper we report on our efforts to repackage analytical solutions, by adding information and verified algorithms to make them better suited for engineering applications, and to make these widely available. The contributions of this paper include: introduction of a database for heat conduction that we call the EXact Analytical Conduction Toolbox, or EXACT; a review of intrinsic verification which is a concept applied to every solution, algorithm, and computer program in the database; and, examples of solutions for semi-infinite and layered bodies undergoing transient heating. One application for these solutions is for the flash-diffusivity method of thermal property measurement.

Database for heat conduction

Many analytical solutions already exist in the literature, for example, Carslaw and Jaeger [1] provide a very large number of solutions. However locating an existing solution solution can be difficult. Thambynayagam [2] recently produced a monumental handbook containing over one thousand transient diffusion solutions. To some extent Thambynayagam solved the index problem by using sketches of the geometries with the boundary conditions indicated by a D, N or R for conditions of the 1^{st}, 2^{nd} and 3^{rd} kinds. This helps considerably but sketches cannot be readily indexed. Polyanin [3] has provided an extensive handbook of solutions of linear partial differential equations, including the diffusion equation, and this handbook is organized by the type of partial differential equation. Since the fields of heat conduction and diffusion are relatively mature and also have a great many exact solutions, it is desirable to catalog and index the solutions available in Carslaw and Jaeger, Polyanin, Thambynayagam and others. However, to make the solutions more accessible and reduce redundant new derivations, a numbering system is needed and is recommended. It is one of the reasons for this paper.

The use of a numbers to identify boundary conditions of the first, second and third kinds of boundary conditions have been in common usage for many years (see for example [4]). We use a numbering system to provide a means to store and retrieve solutions through a unique location in a database. The numbering system used here was introduced in 1988 [5].

Intrinsic verification

Intrinsic verification is the concept that an analytical solution contains within itself the means for checking that numerical results from the solution are correct to high precision. There are several methods of intrinsic verification that have been used to check solutions included in the EXACT database. The methods of intrinsic verification include: use of two independent solutions; time partitioning; additive components; penetration time; and, deviation time. A complete discussion of intrinsic verification is given elsewhere [7].

Examples

In the conference presentation, several examples will be given of solutions that are available at the internet site EXact Analytical Conduction Toolbox, or EXACT. Two of the solutions have been published previously, and other examples are new to the literature. The purpose of these examples is to demonstrate the type of added information available for each solution documented at the EXACT website.

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